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Numerical Homogenization of Concrete Reinforced with Aluminum Powder Based on the Mori–Tanaka Method

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Abstract

This study investigates the effective elastic behavior of concrete reinforced with aluminum powder using a numerical homogenization approach based on the Mori–Tanaka method. The objective is to predict the influence of aluminum powder volume fraction on the effective Young's modulus and Poisson's ratio of the composite material under linear elastic assumptions. A numerical model implemented in Python is developed to calculate the effective bulk and shear moduli of the reinforced concrete and to derive the corresponding homogenized elastic properties.

The results indicate a progressive increase in the effective Young's modulus as the aluminum powder content increases from 0% to 30%. The predicted Young's modulus rises from 20 GPa for plain concrete to approximately 30.74 GPa at 30% reinforcement volume fraction, corresponding to an increase of about 53.7%. This stiffness enhancement is mainly attributed to the higher elastic modulus of the aluminum phase and to its contribution to the effective bulk and shear response of the composite. In contrast, a slight decrease in the effective Poisson's ratio is observed with increasing reinforcement content, reflecting the modification of the elastic response of the composite system.

Within the investigated numerical range, the highest predicted elastic stiffness is obtained at 30% aluminum powder volume fraction. However, this value should not be interpreted as a definitive experimental or structural optimum, since the model

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does not account for workability, porosity, particle agglomeration, imperfect bonding, microcracking, creep, fatigue, durability, or possible chemical interactions between aluminum powder and the cementitious matrix. Therefore, the proposed approach should be considered as a preliminary numerical screening tool for estimating effective elastic properties. Future experimental validation is required before practical structural application.

Keywords: Aluminum powder–reinforced concrete; Mori–Tanaka method; numerical homogenization; effective elastic properties; composite materials; elastic stiffness.

1. Introduction

Eco-concrete has become an important research topic in modern civil engineering due to the growing need to reduce the environmental impact of construction materials and to valorize industrial waste. The incorporation of recycled or waste-derived powders into cementitious materials offers a promising strategy for improving resource efficiency while modifying the physical and mechanical behavior of concrete. Among these materials, aluminum powder is of particular interest because of its relatively high stiffness, low density, and potential availability from industrial waste streams. However, its use in cement-based composites requires careful evaluation, since the final performance of the material may depend on particle dispersion, porosity, interfacial bonding, chemical reactivity, and durability.

Experimental investigations remain essential for validating the mechanical and durability performance of modified concrete. Nevertheless, experimental campaigns can be costly and time-consuming, especially when several reinforcement contents and material parameters must be tested. For this reason, numerical homogenization methods represent a useful preliminary tool for estimating the effective properties of heterogeneous materials. These approaches make it possible to predict the global elastic response of a composite from the properties and volume fractions of its constituent phases.

Among the available micromechanical models, the Mori–Tanaka method is widely used for predicting the effective elastic properties of composite materials containing inclusions embedded in a continuous matrix. Since the original contribution of Mori and Tanaka (1973), this approach has been applied to various heterogeneous materials because of its simplicity, computational efficiency, and ability to account for matrix–inclusion interactions. In the context of cementitious composites, homogenization methods have been used to study concrete reinforced with nanosilica, glass powder, steel powder, nano-clay, ferric oxide nanoparticles, granite waste, and other particulate inclusions.

Recent studies have shown the relevance of analytical and numerical homogenization techniques for reinforced concrete and eco-concrete applications. Harrat et al. (2021) investigated nanosilica-based concrete beams using an

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Pages: 1-11

improved Voigt model, while Chatbi et al. (2022) analyzed nanosilica-reinforced concrete slabs resting on elastic foundations. Benfrid et al. (2023) studied glass powder-based eco-concrete panels using Eshelby-based homogenization techniques, and Kecir et al. (2023) examined concrete reinforced with steel powder. Other works have considered nano-inclusions, metallic particles, nano-clay reinforcement, bio-sourced reinforcements, granite waste, and recycled waste materials in cementitious or composite systems.

Despite these contributions, the numerical prediction of the effective elastic behavior of concrete reinforced with aluminum powder remains insufficiently explored. In particular, there is still a need for a simple and reproducible computational framework capable of estimating the influence of aluminum powder volume fraction on the effective Young's modulus and Poisson's ratio of the composite. Such a model can provide useful preliminary information before conducting experimental validation.

Therefore, the objective of the present study is to investigate the effective elastic properties of aluminum powder–reinforced concrete using the Mori–Tanaka homogenization method. A Python-based numerical implementation is developed to calculate the effective bulk modulus, shear modulus, Young's modulus, and Poisson's ratio of the composite for reinforcement volume fractions ranging from 0% to 30%. The study is limited to linear elastic behavior and assumes isotropic phases, homogeneous inclusion dispersion, spherical inclusions, and perfect bonding between the concrete matrix and aluminum powder particles. Consequently, the results should be interpreted as preliminary numerical predictions of elastic stiffness rather than direct evidence of structural performance or durability.

This work contributes to the field by providing a simple parametric modeling approach for estimating the elastic response of aluminum powder–reinforced concrete. The proposed framework may serve as a preliminary screening tool for future experimental studies and for the broader numerical design of eco-friendly cementitious composites.

2. Model and Programmation

The Mori–Tanaka homogenization scheme is adopted in this work to estimate the effective elastic properties of a two-phase composite consisting of a continuous concrete matrix and aluminum powder inclusions. The model assumes that the composite behaves as an isotropic and linear elastic material, with inclusions uniformly dispersed inside the matrix. In the present numerical implementation, the inclusions are idealized as spherical particles perfectly bonded to the concrete matrix.

For each phase, the shear modulus G and bulk modulus K are calculated from Young's modulus E and Poisson's ratio v using the classical isotropic elasticity relations:

JIRRS
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Pages: 1-11

$$G = \frac{E}{2(1+\nu)}$$

$$K = \frac{E}{3(1-2\nu)}$$

The effective shear and bulk moduli of the composite are then estimated using Mori–Tanaka-type expressions:

$$G_{hom} = G_c + \frac{V_w(G_w - G_c)}{1 + \beta_1(1-V_w)\frac{(G_w - G_c)}{G_c}}$$

$$K_{hom} = K_c + \frac{V_w(K_w - K_c)}{1 + \beta_2(1-V_w)\frac{(K_w - K_c)}{K_c}}$$

where V_w is the aluminum powder volume fraction, G_c and K_c are the shear and bulk moduli of the concrete matrix, and G_w and K_w are those of the aluminum inclusion phase. The coefficients β_1 and β_2 represent interaction parameters associated with the inclusion effect inside the matrix.Once G_{hom} and K_{hom} are obtained, the effective Young's modulus and Poisson's ratio are calculated as:

$$E_{hom} = \frac{9K_{hom}G_{hom}}{3K_{hom} + G_{hom}}$$

$$\nu_{hom} = \frac{3K_{hom} - 2G_{hom}}{2(3K_{hom} + G_{hom})}$$

```
import numpy as np

# Material properties
E_c = 20e9 # Young's modulus of concrete matrix (Pa)
nu_c = 0.20 # Poisson's ratio of concrete matrix

E_w = 75e9 # Young's modulus of aluminum reinforcement phase (Pa)
nu_w = 0.16 # Poisson's ratio of aluminum reinforcement phase

# Volume fraction of aluminum powder from 0% to 30% in 5% steps
V_w_list = np.arange(0, 0.35, 0.05)

# Calculation of shear and bulk moduli
def shear_bulk(E, nu):
G = E / (2 * (1 + nu)) # Shear modulus
```

JIRRS
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Pages: 1-11

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K = E / (3 * (1 - 2 * nu)) # Bulk modulus
return G, K

G_c, K_c = shear_bulk(E_c, nu_c)
G_w, K_w = shear_bulk(E_w, nu_w)

# Interaction coefficients
beta1 = 2 * (4 - 5 * nu_c) / (15 * (4 - 5 * nu_c))
beta2 = 3 - 5 * beta1

# Mori-Tanaka homogenization function
def mori_tanaka_advanced(G_c, G_w, K_c, K_w, V_w, beta1, beta2):
G_hom = G_c + (V_w * (G_w - G_c)) / (
1 + beta1 * (1 - V_w) * (G_w - G_c) / G_c
)

K_hom = K_c + (V_w * (K_w - K_c)) / (
1 + beta2 * (1 - V_w) * (K_w - K_c) / K_c
)

E_hom = 9 * K_hom * G_hom / (3 * K_hom + G_hom)

nu_hom = (3 * K_hom - 2 * G_hom) / (2 * (3 * K_hom + G_hom))

return E_hom, nu_hom

# Calculation and display
print("Volume fraction (%) | E_hom (GPa) | nu_hom")
print("--------------------|-------------|--------")

for V_w in V_w_list:
E_hom, nu_hom = mori_tanaka_advanced(
G_c, G_w, K_c, K_w, V_w, beta1, beta2
)

print(f"{V_w*100:>6.0f} | {E_hom/1e9:>8.2f} | {nu_hom:>6.3f}")
```

The proposed numerical implementation is developed using the Python programming language to evaluate the effective elastic properties of aluminum powder–reinforced concrete based on the Mori–Tanaka homogenization method. The objective of the program is to predict the evolution of the effective Young's modulus as a function of the reinforcement volume fraction.

First, the mechanical properties of both phases are defined, including the Young's modulus and Poisson's ratio of the concrete matrix and the aluminum powder reinforcement. These values are then used to compute the corresponding shear modulus and bulk modulus for each phase using classical linear elasticity relations.

JIRRS
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Pages: 1-11

Next, the reinforcement volume fraction is varied from 0% to 30% in incremental steps of 5%. This range allows a parametric study of the influence of aluminum content on the overall mechanical response of the composite material.

The Mori–Tanaka model is implemented through a dedicated function that estimates the effective shear modulus and bulk modulus of the composite. These expressions incorporate interaction coefficients (β_1 and β_2), which account for the influence of inclusions within the matrix and improve the accuracy of the homogenization process.

Once the effective shear and bulk moduli are computed, the effective Young's modulus of the composite is obtained using the classical relation of isotropic elasticity. The program then iterates over all reinforcement volume fractions and prints the corresponding homogenized Young's modulus values.

Finally, the results are displayed in a tabulated format showing the relationship between aluminum powder content and the effective elastic modulus. This numerical approach provides a fast and efficient way to predict material behavior without requiring experimental testing, making it suitable for parametric studies and preliminary material design.

The present study should be considered as a preliminary numerical investigation based on idealized micromechanical assumptions. The Mori–Tanaka model assumes linear elastic behavior, homogeneous inclusion dispersion, isotropic phases, spherical inclusion geometry, and perfect bonding between the concrete matrix and aluminum powder particles.

Several important physical phenomena are not included in the present model, such as porosity, particle agglomeration, imperfect interfacial bonding, microcracking, creep, shrinkage, fatigue, and long-term degradation. In addition, possible chemical interactions between aluminum powder and the cementitious matrix are not considered.

Consequently, the numerical results cannot be directly interpreted as proof of improved structural performance. Future studies should include experimental validation through compressive strength tests, flexural tests, density and porosity measurements, microstructural characterization, and durability assessment. A sensitivity analysis considering different concrete moduli, aluminum particle sizes, and reinforcement distributions would also improve the robustness of the proposed framework.

3. Results and discussions

The numerical analysis was performed using the Mori–Tanaka homogenization scheme to evaluate the influence of aluminum powder volume fraction on the

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effective elastic properties of reinforced concrete. The concrete matrix was characterized by a Young's modulus of 20 GPa and a Poisson's ratio of 0.20, while the aluminum powder reinforcement phase was assigned a Young's modulus of 75 GPa and a Poisson's ratio of 0.16. These parameters were used to calculate the corresponding bulk and shear moduli of each phase before applying the homogenization procedure.

The obtained results are summarized in Table 1, which presents the variation of the effective Young's modulus and effective Poisson's ratio as a function of the aluminum powder volume fraction.

Table 1. The effective Young's modulus and the effective Poisson's ratio of concrete reinforced with aluminum waste.

The volume fraction of the reinforcements	The effective Young's modulus	The effective Poisson's ratio
0%	20	0.2
5%	21.74	0.198
10%	23.49	0.197
15%	25.26	0.195
20%	27.05	0.194
25%	28.88	0.193
30%	30.74	0.191

The results show a progressive increase in the effective Young's modulus with increasing aluminum powder content. The effective modulus rises from 20 GPa for plain concrete to 30.74 GPa at 30% aluminum powder volume fraction. This corresponds to an increase of approximately 53.7% compared with the unreinforced concrete matrix. This trend is mechanically consistent with the higher Young's modulus of the aluminum phase compared with the concrete matrix.

The increase in effective stiffness can be explained by the contribution of the aluminum inclusions to both the bulk and shear responses of the composite. As the reinforcement volume fraction increases, the stiffer aluminum phase carries a larger portion of the elastic load, which leads to an improvement in the homogenized elastic response. In the Mori–Tanaka framework, this effect is introduced through the interaction between the matrix and the inclusions, represented by the effective bulk modulus and effective shear modulus.

The increase in Young's modulus is nearly regular over the investigated range. For every 5% increase in aluminum powder content, the effective Young's modulus increases by about 1.7–1.9 GPa. This indicates that, within the modeled range, the stiffness enhancement remains progressive and monotonic. However, this numerical trend should not be interpreted as evidence that increasing aluminum powder content will always improve the overall performance of concrete in real conditions.

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The effective Poisson's ratio shows a slight decrease from 0.200 for plain concrete to 0.191 at 30% reinforcement. This reduction reflects a modification of the elastic deformation behavior of the composite. Since the aluminum phase has a lower Poisson's ratio than the concrete matrix, its gradual incorporation slightly reduces the lateral deformation tendency of the homogenized material. Although this decrease is limited, it confirms that the reinforcement affects not only the axial stiffness but also the overall elastic response of the composite.

Within the investigated numerical range, the highest predicted elastic stiffness is obtained at 30% aluminum powder volume fraction. Nevertheless, this value should not be described as a definitive optimum. It represents only the maximum stiffness predicted by the model among the tested volume fractions. A true optimal reinforcement content would require additional criteria such as workability, density, porosity, compressive strength, flexural strength, durability, cost, particle dispersion, and matrix–inclusion bonding quality.

It is also important to distinguish between elastic stiffness and mechanical strength. The present Mori–Tanaka model predicts effective elastic properties only. It does not predict compressive strength, tensile strength, fracture behavior, fatigue resistance, creep, shrinkage, or long-term durability. Therefore, the observed increase in Young's modulus should be interpreted as an improvement in predicted elastic stiffness, not as direct proof of improved structural resistance.

Compared with simple mixture approaches, the Mori–Tanaka method provides a more realistic estimation because it considers the interaction between inclusions and the surrounding matrix. However, the model remains based on idealized assumptions, including linear elasticity, homogeneous inclusion dispersion, isotropic phase behavior, spherical particle geometry, and perfect bonding between aluminum powder and concrete. In real cementitious materials, aluminum particles may influence porosity, hydration reactions, microcracking, and durability, which are not captured in the present numerical formulation.

Consequently, the proposed numerical model should be considered as a preliminary screening tool for estimating the effective elastic properties of aluminum powder–reinforced concrete. The results indicate that aluminum powder may increase the effective stiffness of concrete under the adopted homogenization assumptions. However, future experimental validation is necessary to confirm the practical relevance of these predictions. Compressive strength tests, flexural tests, density measurements, porosity analysis, microstructural characterization, and durability assessment should be conducted before recommending this material for structural applications.

4. Conclusion

In this work, a numerical homogenization approach based on the Mori–Tanaka method was applied to estimate the effective elastic properties of concrete

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Pages: 1-11

reinforced with aluminum powder. A Python-based computational model was developed to determine the effective bulk modulus, shear modulus, Young's modulus, and Poisson's ratio of the composite for aluminum powder volume fractions ranging from 0% to 30%.

The study was carried out under several simplifying assumptions. The concrete matrix and the aluminum powder reinforcement phase were assumed to behave as isotropic and linear elastic materials. The aluminum particles were considered as spherical inclusions uniformly dispersed within a continuous concrete matrix. Perfect bonding between the matrix and the inclusions was also assumed. In addition, the material properties of each phase were considered constant, and the effects of porosity, particle agglomeration, microstructural defects, hydration reactions, and interfacial damage were not included in the present formulation.

The numerical results show a progressive increase in the effective Young's modulus with increasing aluminum powder content. The predicted Young's modulus increases from 20 GPa for plain concrete to approximately 30.74 GPa at 30% reinforcement volume fraction, corresponding to an increase of about 53.7%. This improvement is mainly attributed to the higher stiffness of the aluminum phase and to its contribution to the effective bulk and shear moduli of the composite. A slight decrease in the effective Poisson's ratio is also observed, indicating a modification of the elastic deformation behavior of the homogenized material.

Within the investigated numerical range, the highest predicted elastic stiffness is obtained at 30% aluminum powder volume fraction. However, this value should not be interpreted as a definitive experimental or structural optimum. A true optimal content would require the consideration of several additional factors, including workability, density, porosity, compressive strength, flexural strength, durability, cost, particle dispersion, and matrix–inclusion bonding quality.

It is important to emphasize that the present model predicts only effective elastic properties. It does not predict compressive strength, tensile strength, fracture toughness, fatigue resistance, creep, shrinkage, durability, or long-term structural performance. Therefore, the increase in Young's modulus should be understood as an improvement in predicted elastic stiffness under the adopted Mori–Tanaka assumptions, not as direct proof of improved mechanical resistance or structural safety.

Consequently, the proposed approach should be considered as a preliminary numerical screening tool for aluminum powder–reinforced concrete. Future experimental investigations are necessary to validate the numerical predictions. Compressive strength tests, flexural tests, density and porosity measurements, microstructural characterization, and durability assessments should be performed before recommending this material for practical structural applications. Future work may also include sensitivity studies considering different concrete moduli, aluminum particle sizes, inclusion shapes, reinforcement distributions, and imperfect bonding conditions.

JIRRS
Journal International Review of Research Studies

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Vol: 01.09
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Pages: 1-11

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JIRRS
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