

Analytical Investigation of Thermomechanical Stability in High-Performance Eco-Concrete Panels Incorporating Waste Glass Powder: A Linear Thermoelastic Plate-Buckling Approach

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Abstract

The increasing demand for sustainable construction materials has encouraged the use of recycled industrial and municipal waste in concrete technology. Waste glass powder (WGP) is considered a promising supplementary cementitious material because it may contribute to cement reduction and waste valorization. This study investigates the thermomechanical buckling behavior of thin high-performance eco-concrete panels incorporating WGP under uniform thermal loading. A linear thermoelastic analytical formulation based on classical small-deflection plate theory is adopted to determine the critical buckling temperature difference, ΔT_{cr} , of simply supported rectangular panels. The analysis considers the influence of the width-to-length ratio, b/a , thickness-to-length ratio, h/a , Poisson's ratio, and WGP content. In the present model, the effect of WGP is represented through Young's modulus and Poisson's ratio, while the thermal expansion coefficient is assumed constant. The results show that ΔT_{cr} decreases as the panel becomes thinner and as the b/a ratio increases. For the reference case, ΔT_{cr} decreases from 156.6°C to 87.1°C for $h/a=1/30$, and from 69.6°C to 38.7°C for $h/a=1/45$, when b/a increases from 1 to 3. The incorporation of WGP produces a limited but consistent increase in thermal buckling resistance. For square panels, high-performance concrete containing 20% and 30% WGP shows slightly higher critical temperatures than the reference high-performance concrete, mainly due to the reduction in Poisson's ratio. These findings indicate that WGP-modified high-performance concrete may offer a moderate improvement in the thermal buckling stability of idealized thin simply supported panels. However, the conclusions remain limited to the assumptions of linear thermoelastic behavior, homogeneous material properties, constant thermal expansion coefficient, and uniform thermal loading.

Keywords: Waste glass powder; Eco-concrete; Thermomechanical buckling; Critical temperature; High-performance concrete; Linear thermoelastic analysis.

1. Introduction

Concrete is one of the most widely used construction materials worldwide and continues to serve as the foundation of modern infrastructure, including buildings, bridges, highways, tunnels, and dams [1]. Its widespread application is primarily attributed to its high mechanical strength, durability, versatility, and relatively low cost. However, the growing demand for concrete has raised significant environmental concerns due to the intensive use of Portland cement, whose production is responsible for a substantial share of global carbon dioxide (CO₂) emissions [2]. In addition, the extraction of raw materials and the high energy consumption associated with cement manufacturing contribute to environmental degradation and resource depletion [3]. Consequently, the development of sustainable and environmentally responsible construction materials has become a major objective of contemporary research.

Among the various strategies proposed to reduce the environmental footprint of concrete, the incorporation of recycled waste materials has emerged as a promising approach. Waste glass powder (WGP) has attracted particular interest because of its widespread availability and pozzolanic characteristics, which enable its use as a partial cement replacement in concrete mixtures [4]. The incorporation of WGP may contribute to reducing landfill waste while decreasing cement consumption, thereby supporting more sustainable concrete production practices.

To address the environmental challenges associated with conventional concrete production, numerous experimental, analytical, and numerical investigations have been conducted on recycled and alternative materials incorporated into cementitious composites. These studies have focused on enhancing the mechanical, thermal, and durability performance of concrete while promoting sustainable construction practices.

Safi et al. [5] investigated the compressive and flexural behavior of concrete containing different percentages of waste glass powder and reported that a replacement ratio of approximately 17.5% provided the most favorable mechanical performance. Ahmad et al. [6] demonstrated that waste glass powder improves mortar strength through pozzolanic reactions and highlighted the effectiveness of predictive approaches for evaluating concrete properties. Matos et al. [7] studied sustainable high-performance concrete incorporating waste glass powder and limestone filler and showed that optimized mixtures can improve mechanical performance while reducing environmental impacts. Similarly, Salahaddin et al. [8] reported improvements in the microstructure and compressive strength of ultra-high-performance concrete containing waste glass powder, whereas Dai et al. [9] demonstrated its effectiveness in enhancing the high-temperature stability of cement-based materials.

In parallel, several analytical studies have examined the thermo-mechanical behavior of concrete structures reinforced with recycled materials and nanoparticles. Benfrid et al. [10] analyzed the thermal buckling behavior of eco-concrete slabs reinforced with bovine bone particles and highlighted the potential of recycled biomaterials for improving thermal stability. Kecir et al. [11] investigated

concrete slabs reinforced with Fe_2O_3 nanoparticles using a quasi-three-dimensional shear deformation theory and reported significant improvements in stiffness and reductions in deflection. Dine Elhennani et al. [12] studied the buckling and free vibration behavior of nanoparticle-reinforced concrete beams and demonstrated the beneficial influence of nanoparticles on structural performance.

Particular attention has also been devoted to concrete systems incorporating recycled glass powder and nanomaterials. Benfrid et al. [13] investigated the thermomechanical response of concrete panels containing waste glass powder and demonstrated that glass powder content, geometric parameters, and thermal loading conditions influence structural behavior. Chatbi et al. [14] examined the static response of concrete beams reinforced with nano-clay platelets and showed that nano-reinforcement can substantially improve stiffness and load-carrying capacity. Harrat et al. [15] developed a refined trigonometric shear deformation model for thermoelastic concrete slabs reinforced with ferric oxide nanoparticles and reported improvements in thermo-mechanical performance. Furthermore, Chatbi et al. [16] studied the bending behavior of silica nanoparticle-reinforced concrete slabs resting on elastic foundations and highlighted the beneficial effect of nanoparticles on structural rigidity. In another contribution, Chatbi et al. [17] analyzed carbon nanotube-reinforced composite beams and confirmed that nanomaterials significantly improve vibration characteristics and structural stability.

Despite the considerable progress achieved in the development of recycled and nano-reinforced concrete materials, most existing studies have primarily focused on mechanical strength, durability, microstructural evolution, vibration behavior, or general thermo-mechanical responses. Comparatively limited attention has been devoted to the thermal buckling instability of thin concrete panels incorporating waste glass powder. Moreover, the specific influence of WGP-induced variations in elastic properties on the critical buckling temperature remains insufficiently understood. Consequently, further analytical investigations are required to clarify the relationship between material-property modifications induced by WGP and the thermomechanical stability of concrete panels.

Thermal buckling represents a critical instability phenomenon that may occur in thin structural elements exposed to elevated temperatures. Understanding the influence of recycled materials on thermal stability is therefore important for the design of durable and resilient concrete systems. However, the available literature remains limited regarding the thermal buckling response of high-performance concrete panels incorporating waste glass powder under uniform thermal loading.

The novelty of the present study lies in the application of a linear thermoelastic analytical formulation to investigate the thermal buckling behavior of thin rectangular eco-concrete panels containing waste glass powder. Unlike previous investigations, the present work specifically examines the influence of WGP-induced variations in elastic properties on the critical buckling temperature difference and provides a detailed assessment of the effects of geometric and material parameters on thermomechanical stability.

The present study investigates the thermomechanical buckling behavior of thin rectangular eco-concrete panels incorporating waste glass powder. A linear thermoelastic analytical model based on the classical theory of small elastic deformations is employed to determine the critical buckling temperature difference (ΔT_{cr}) under uniform thermal loading. The effects of the width-to-length ratio (b/a), thickness-to-length ratio (h/a), Poisson's ratio, initial temperature, and waste glass powder content are systematically examined. Ordinary concrete (OC), high-performance concrete (HPC), and high-performance concrete containing waste glass powder (HPC-WGP) are considered. The primary objective is to evaluate the influence of waste glass powder on the thermal buckling resistance of thin simply supported panels through its effect on the elastic properties of the material. In addition, the study contributes to the broader effort toward sustainable construction through the valorization of waste glass and the partial reduction of cement consumption.

2. Mathematical model

For the sake of clarity, the complete sequence of calculations adopted in the present work is summarized in the flowchart shown in Figure 1. This diagram provides a comprehensive overview of the analytical procedure, beginning with the definition of geometric and material parameters, followed by the determination of displacement, strain, and stress fields, the formulation of equilibrium equations, the application of boundary conditions, and finally the calculation of the critical thermal buckling temperature difference.

The analytical formulation adopted in the present study is based on the thermoelastic buckling theory of thin rectangular plates proposed by Cheng [18] and adapted to high-performance eco-concrete panels incorporating waste glass powder (WGP). The model relies on the classical Kirchhoff–Love plate theory developed by Ventsel and Krauthammer [19] together with the fundamental principles of continuum mechanics. The analysis is performed within the framework of linear thermoelasticity and small-deflection theory, which is commonly employed for the stability analysis of thin plate structures.

The mathematical model is established under the assumptions of homogeneous and isotropic material behavior, small displacements, linear strain–displacement relations, and simply supported boundary conditions along all panel edges. According to the Kirchhoff–Love assumptions, transverse shear deformation is neglected, and normal lines to the middle surface remain straight and perpendicular to the deformed mid-surface after deformation [19]. These assumptions are considered appropriate for the analysis of thin concrete panels.

The displacement field is first defined using the classical plate kinematics. The corresponding strain components are subsequently derived from the geometric relations of continuum mechanics. The stress field is then obtained through Hooke's law while accounting for the thermal strains induced by a uniform temperature

increase. In the present formulation, the influence of waste glass powder is represented through experimentally determined values of Young's modulus and Poisson's ratio. The coefficient of thermal expansion is assumed constant throughout the analysis.

The membrane forces and bending moments are calculated by integrating the stress components through the panel thickness. These force and moment resultants are subsequently introduced into the governing equilibrium equations of thin plates following the formulations reported by Han et al. [23], Gerard [24], and Wu [25]. The resulting differential equation governs the thermoelastic stability behavior of the eco-concrete panel subjected to thermal loading.

To determine the buckling condition, a uniform temperature increase is applied over the entire panel surface, generating compressive thermal membrane forces. The simply supported boundary conditions are then imposed, and Navier's analytical solution method is employed to obtain a closed-form solution for the displacement field [19]. This procedure leads to the determination of the critical thermal buckling temperature difference (ΔT_{cr}), which corresponds to the temperature increase at which the panel reaches its stability limit and buckling occurs.

The thermo-mechanical response of the panel is therefore evaluated as a function of its geometric characteristics and elastic material properties. Particular attention is devoted to the influence of the width-to-length ratio (b/a), thickness-to-length ratio (h/a), Poisson's ratio, and waste glass powder content on the critical buckling temperature. The adopted formulation enables the assessment of the relative contribution of these parameters to the thermomechanical stability of eco-concrete panels.

It should be noted that the present analytical model is based on linear thermoelastic behavior and does not explicitly account for cracking, creep, shrinkage, reinforcement effects, moisture migration, or temperature-dependent material degradation. Furthermore, the thermal expansion coefficient is assumed constant, and the analysis is limited to thin homogeneous panels subjected to uniform thermal loading and simply supported boundary conditions. Consequently, the obtained results should be interpreted within the limitations associated with these assumptions.

For clarity, the complete sequence of calculations adopted in the present work is summarized in the flowchart shown in Figure 1. The flowchart provides an overview of the analytical procedure, beginning with the definition of geometric and material parameters, followed by the determination of displacement, strain, and stress fields, the formulation of equilibrium equations, the application of boundary conditions, and finally the calculation of the critical thermal buckling temperature difference (ΔT_{cr}).

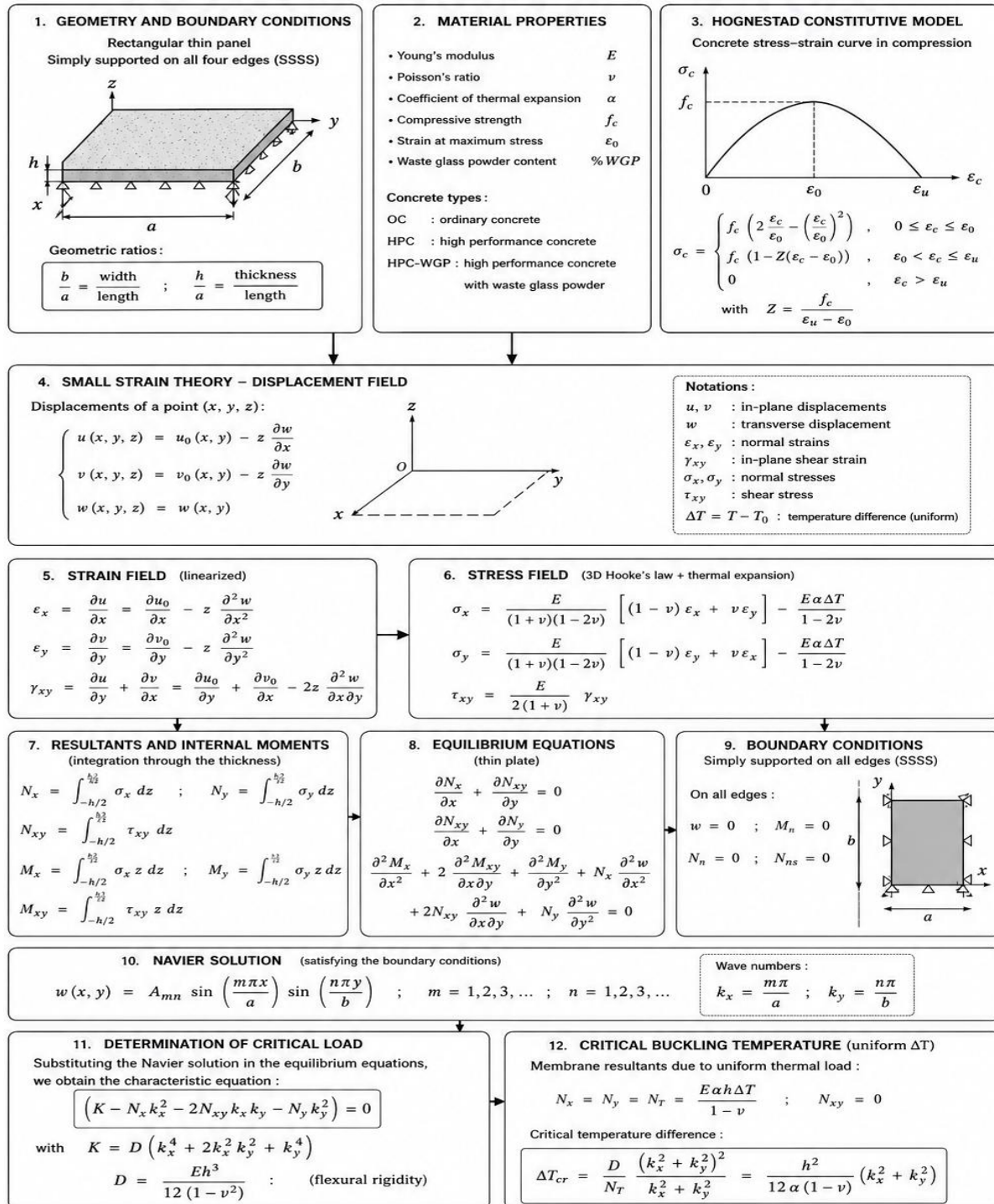


Figure 1. Analytical flowchart for determining the critical thermal buckling temperature difference (ΔT_{cr}) of WGP-modified eco-concrete panels.

3. Results and discussions

The validity of the present analytical model was assessed by comparing the predicted critical buckling temperature difference, ΔT_{cr} , with the benchmark results reported by Cheng (2018) [20]. As shown in Figure 2, good agreement was obtained for all investigated thickness ratios, namely $h/a=1/30$, $1/35$, $1/40$, and $1/45$. This comparison confirms that the adopted formulation correctly reproduces the classical thermoelastic buckling response of thin rectangular plates under uniform thermal loading.

However, this validation should be interpreted with caution because it is limited to comparison with an analytical benchmark solution. It validates the mathematical implementation of the plate-buckling formulation, but it does not constitute an experimental validation of WGP-modified concrete panels. Therefore, further numerical and experimental investigations are required to confirm the applicability of the model to real concrete elements.

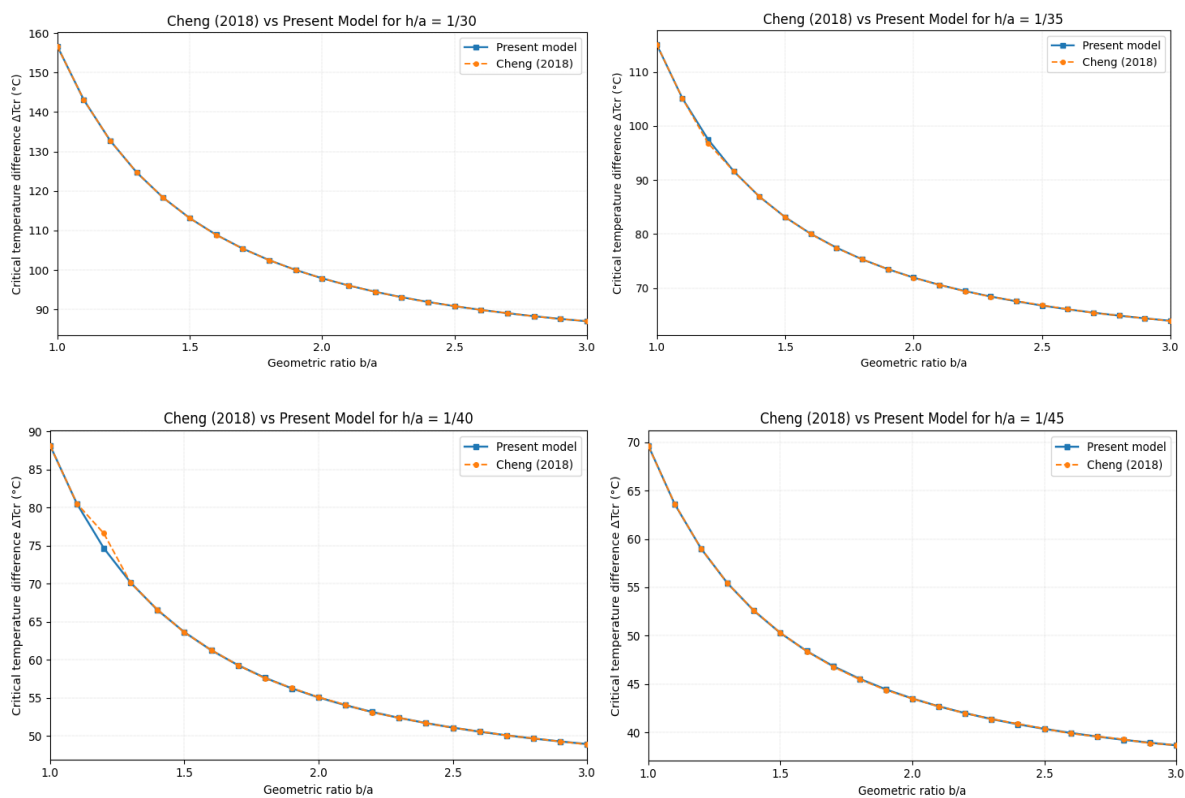


Figure 2. Validation of the present analytical model by comparison with Cheng's (2018) [20] benchmark solution for different thickness ratios ($h/a=1/30$, $1/35$, $1/40$, $1/45$).

To investigate the influence of waste glass powder (WGP) on the thermomechanical buckling behavior of high-performance concrete (HPC) panels, the experimental data reported by Abdenour et al. (2017) [27] were used to introduce variations in the elastic material properties. Three high-performance concrete mixtures cured for 28 days were considered: conventional HPC without glass powder, HPC containing 20% glass powder as a partial cement replacement, and HPC containing 30% glass

powder. The corresponding cement dosages were 420, 336, and 294 kg/m³, while the glass powder contents were 0, 84, and 126 kg/m³, respectively. The measured Young's moduli were 45.037 GPa for HPC without WGP, 41.13 GPa for HPC with 20% WGP, and 38.9 GPa for HPC with 30% WGP. Similarly, Poisson's ratio decreased from 0.22 to 0.21 and 0.19 with increasing WGP content. In the present analytical model, the effect of WGP is therefore represented only through Young's modulus and Poisson's ratio. The coefficient of thermal expansion is assumed constant and equal to $\alpha=1\times 10^{-5}\text{ }^{\circ}\text{C}^{-1}$.

Based on these experimentally determined elastic properties, a parametric analysis was performed to evaluate ΔT_{cr} under uniform thermal loading. The corresponding numerical results are presented in Table 5 and Figure 3. This analysis allows the influence of WGP incorporation, panel thickness, and aspect ratio on the thermal buckling stability of high-performance eco-concrete panels to be assessed. Figure 2 shows that ΔT_{cr} decreases as b/a increases from 1 to 3 for all investigated thickness ratios. This trend indicates that elongated panels are more sensitive to thermal buckling than square panels. The figure also shows that thicker panels exhibit higher critical temperatures due to their greater flexural rigidity, whereas thinner panels present lower thermal stability. The small differences between the present results and Cheng's benchmark values are mainly attributed to rounding and numerical approximation..

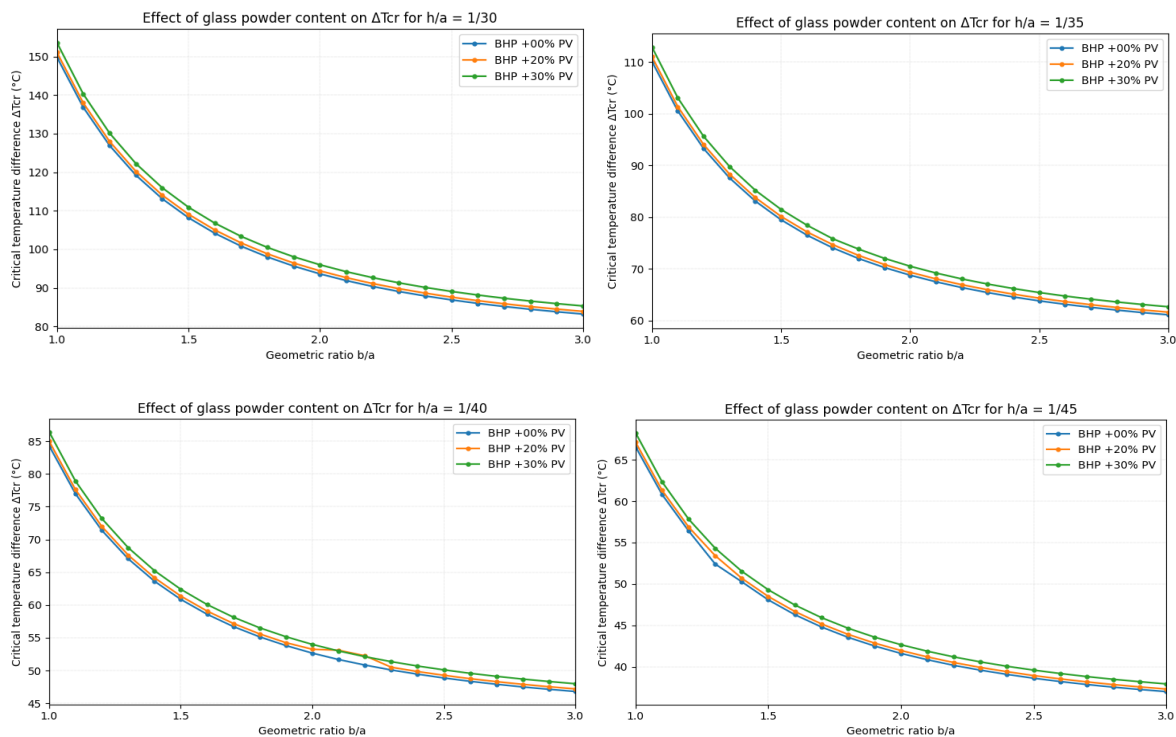


Figure 3. Effect of WGP content and geometric parameters on the critical buckling temperature difference, ΔT_{cr} , of high-performance eco-concrete panels.

Figure 3 illustrates the combined influence of WGP content and geometric parameters on ΔT_{cr} . A decreasing trend is observed as b/a increases from 1 to 3, regardless of the WGP content or thickness ratio. This confirms that panel geometry has a dominant influence on thermal buckling resistance. In particular, the thickness-to-length ratio plays a major role, since panels with $h/a=1/30$ exhibit the highest critical temperatures, whereas panels with $h/a=1/45$ show the lowest values.

A comparison between the different concrete mixtures indicates that the incorporation of WGP produces a limited but consistent increase in the critical buckling temperature. For example, at $b/a=1$ and $h/a=1/30$, ΔT_{cr} increases from approximately 149.81°C for HPC without WGP to 151.04°C and 153.58°C for mixtures containing 20% and 30% WGP, respectively. These values correspond to increases of approximately 0.82% and 2.52%, respectively. Therefore, the improvement should be described as moderate rather than significant.

This limited increase can be mainly attributed to the reduction in Poisson's ratio associated with WGP incorporation. Although Young's modulus decreases as the WGP content increases, the reduction in Poisson's ratio contributes to a slight increase in ΔT_{cr} within the adopted analytical formulation. Therefore, the influence of WGP should be interpreted as a material-parameter effect rather than as a general proof of enhanced thermomechanical performance.

Overall, the results show that geometric parameters, especially h/a and b/a , have a stronger influence on ΔT_{cr} than WGP content. The incorporation of WGP may contribute to a slight improvement in thermal buckling resistance under the assumptions of the present linear thermoelastic model. Nevertheless, broader claims regarding sustainability and thermomechanical enhancement should be made cautiously, since the present analysis does not include temperature-dependent degradation, cracking, creep, shrinkage, reinforcement effects, moisture migration, or experimental thermal buckling validation.

5. Conclusion

This study analytically investigated the thermomechanical buckling behavior of thin high-performance eco-concrete panels incorporating waste glass powder (WGP). The adopted linear thermoelastic model was validated against Cheng's benchmark solution, showing good agreement and confirming the correct implementation of the analytical plate-buckling formulation.

The results showed that the critical buckling temperature difference, ΔT_{cr} , decreases with increasing width-to-length ratio b/a and decreasing thickness-to-length ratio h/a . This indicates that thinner and more elongated panels are more susceptible to thermal instability, while thicker panels exhibit higher resistance due to their greater flexural rigidity.

The incorporation of WGP produced a limited but consistent increase in ΔT_{cr} . Mixtures containing 20% and 30% WGP exhibited slightly higher critical temperatures than conventional high-performance concrete. This improvement is mainly attributed to the reduction in Poisson's ratio in the adopted analytical formulation, although Young's modulus decreases with increasing WGP content. Therefore, the effect of WGP should be interpreted as moderate rather than significant.

Among the investigated mixtures, the concrete containing 30% WGP showed the highest critical temperature values under the assumptions of the present model. However, the conclusions remain limited to thin, homogeneous, simply supported panels subjected to uniform thermal loading. The model does not include cracking, creep, shrinkage, reinforcement effects, moisture migration, temperature-dependent degradation, or experimental thermal buckling validation.

Overall, WGP-modified high-performance concrete may contribute to a slight improvement in thermal buckling resistance while supporting waste valorization and partial cement reduction. Further experimental and numerical studies are required to confirm its applicability to real concrete structures.

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